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Further Studies in Aesthetic Field Theory IV

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Abstract

We are able to obtain a bounded particle, with no indication of a singularity appearing, in several ways different from our previous papers. For one set of data we find slightly greater structure (more turnabout points) than previously. We discuss some of the properties of ten different sets of data.

1. *Introduction*

In previous papers we obtained (Muraskin, 1973a; Muraskin & Ring, 1973; Muraskin, 1973b) a bounded particle from 'aesthetic' mathematical ideas.

Running the computer for various directions from the origin, we found in all cases, for all our particle solutions, that all the field components monotonically approach zero at a sufficient distance from the origin. We ran the computer out as far as $x = 1800$ in one instance. It would be necessary for additional structure to show up eventually if a solution is to describe a many-particle system.

A possible hypothesis to be made is that the 'vacuum' outside the particle shows all sorts of oscillations in the field components. The idea of a highly agitated 'vacuum' has been suggested by many authors (Bohm, 1962; Nelson, 1966; Kershaw, 1964; Boyer, 1968; de la Pena-Auerbach, 1969; Lanzcos, 1957; Wheeler, 1962; and others).

In this paper we discuss several attempts at trying to improve upon our previous results. We find we can only claim mild success in this regard as we have only slightly greater oscillatory behavior for the field components. We shall limit the discussion to the equations $\Gamma^i_{jk;l} = 0$, $g_{ij;k} = 0$.

2. *Data* 1

The values of g_{ij} are not critical for the point we wish to demonstrate below. We take the following for Γ^i_{jk} at the origin point

$$\begin{aligned} \Gamma^1_{23} &= -4\pi & \Gamma^1_{20} &= 8\pi \\ \Gamma^2_{13} &= 4\pi & \Gamma^2_{10} &= -8\pi \\ \Gamma^3_{01} &= \cdot 1 & \Gamma^3_{02} &= 0 \end{aligned} \tag{2.1}$$

All other Γ^i_{jk} are taken to be zero at the origin. When we ran down the z-axis, only the six components listed above remained non-zero. Γ^1_{23}, Γ^2_{13}, Γ^1_{20}; Γ^2_{10} remained unchanged throughout the entire run. The graph of Γ^3_{01} is given by a cosine curve and Γ^3_{02} is given by a sine curve. The amplitude is $\cdot 1$. The wavelength is $2\pi/\Gamma^2_{13} = \cdot 5$ and the period, obtained from running down the time axis, is $2\pi/\Gamma^1_{20} = \cdot 25$.

Thus, it follows that $\Gamma^i_{jk;l} = 0$ can be used to generate sine and cosine curves when we choose the data at the origin as above. Thus, $\Gamma^i_{jk;l} = 0$ is capable of describing an indefinite number of oscillations for a field component.

The difficulty with the above data is that integrability is not satisfied. This difficulty in extracting the sine and cosine effect from the field theory has been demonstrated on previous occasions (Muraskin, 1970; Muraskin, 1972a) using analytic rather than computer considerations.

3. *Data* 2

We take for Γ^i_{jk} the following:

$$\begin{aligned} \Gamma^1_{23} &= -B_0 & \Gamma^1_{20} &= B_3 & \Gamma^1_{30} &= -B_2 & \Gamma^2_{30} &= B_1 \\ \Gamma^3_{21} &= B_0 & \Gamma^0_{21} &= -B_3 & \Gamma^0_{31} &= B_2 & \Gamma^0_{32} &= -B_1 \\ \Gamma^2_{13} &= B_0 & \Gamma^2_{10} &= -B_3 & \Gamma^3_{10} &= B_2 & \Gamma^3_{20} &= -B_1 \\ \Gamma^1_{32} &= B_0 & \Gamma^1_{02} &= -B_3 & \Gamma^1_{03} &= B_2 & \Gamma^2_{03} &= -B_1 \\ \Gamma^2_{31} &= -B_0 & \Gamma^2_{01} &= B_3 & \Gamma^3_{01} &= -B_2 & \Gamma^3_{02} &= B_1 \\ \Gamma^3_{12} &= -B_0 & \Gamma^0_{12} &= B_3 & \Gamma^0_{13} &= -B_2 & \Gamma^0_{23} &= B_1 \end{aligned} \tag{3.1}$$

We also take

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.2}$$

Thus, we have that $g_{im}\Gamma^m_{jk}$ is antisymmetric in all indices at the origin. This set of Γ^i_{jk}, we then find, is preserved at all points by the field equations $\Gamma^i_{jk;l} = 0$. Thus, Γ^i_{jk} are constants and are exact solutions of the field equations. The integrability equations are satisfied but with $R^i{}_{jkl} \neq 0$. Thus, this set of data is a generalisation of the exact Dirac plane wave solution discussed previously (Muraskin, 1971a).

4. *Data* 3

The non-zero components for $\Gamma^\alpha_{\beta\gamma}$ are taken to be

$$\begin{aligned}
&\Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = \Gamma^0_{00} = \Gamma^1_{01} = \Gamma^2_{02} = \Gamma^3_{03} = A \\
&\Gamma^2_{13} = \Gamma^3_{21} = \Gamma^1_{32} = -\Gamma^1_{23} = -\Gamma^3_{12} = -\Gamma^2_{31} = B \\
&\Gamma^0_{11} = \Gamma^0_{22} = \Gamma^0_{33} = -C
\end{aligned} \tag{4.1}$$

$g_{\alpha\beta}$ is taken to be

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.2}$$

This is the same data used in Muraskin (1973b). In Muraskin (1973b) we took $e^\alpha{}_i$ such that g_{00} was a maximum at the origin. In this section we do not impose this requirement and take $e^\alpha{}_i$ to have, for example, the values

$$\begin{array}{llll}
e^1{}_1 = \cdot 88 & e^1{}_2 = -\cdot 42 & e^1{}_3 = -\cdot 32 & e^1{}_0 = \cdot 22 \\
e^2{}_1 = \cdot 5 & e^2{}_2 = \cdot 9 & e^2{}_3 = -\cdot 425 & e^2{}_0 = \cdot 3 \\
e^3{}_1 = \cdot 2 & e^3{}_2 = -\cdot 55 & e^3{}_3 = \cdot 89 & e^3{}_0 = \cdot 6 \\
e^0{}_1 = \cdot 44 & e^0{}_2 = -\cdot 16 & e^0{}_3 = \cdot 39 & e^0{}_0 = 1\cdot 01
\end{array} \tag{4.3}$$

We still found that all components of Γ^i_{jk} and g_{ij} had maximum and minimum behavior in the vicinity of the origin and the values of Γ^i_{jk} and g_{ij} approach zero after we have progressed a sufficient distance from the origin along the axes (as well as on selected runs off the axes). The $e^\alpha{}_i$ above were chosen in no special way. Thus, we get a bounded particle irrespective of the choice of $e^\alpha{}_i$, at least for a wide class of $e^\alpha{}_i$.† The choice of $\Gamma^\alpha_{\beta\gamma}, g_{\alpha\beta}$ are then the significant variables in obtaining a bounded particle here.

5. *Data* 4

In this section we take those $\Gamma^\alpha_{\beta\gamma}$ that describe Dirac plane waves (Muraskin, 1971a) in the x, y, z directions simultaneously, together with those components

† We cannot choose $e^\alpha{}_i$ all zero, for example, and still get a bounded particle.

instrumental in giving a bounded particle. The following $\Gamma^{\alpha}_{\beta\gamma}$ were non-zero

$$\begin{aligned}
&\Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = \Gamma^1_{01} = \Gamma^2_{02} = \Gamma^3_{03} = \Gamma^0_{00} = +1 \\
&\Gamma^0_{11} = \Gamma^0_{22} = \Gamma^0_{33} = -1 \\
&\Gamma^2_{13} = \Gamma^1_{32} = \Gamma^3_{21} = -\Gamma^1_{23} = -\Gamma^2_{31} = -\Gamma^3_{12} = +1 \\
&\Gamma^1_{20} = \Gamma^2_{30} = \Gamma^3_{10} = -\Gamma^1_{30} = -\Gamma^2_{10} = -\Gamma^3_{20} = +1
\end{aligned} \tag{5.1}$$

We also took

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{5.2}$$

and

$$\begin{array}{llll}
e^1{}_1 = \cdot 7 & e^1{}_2 = \cdot 62 & e^1{}_3 = \cdot 46 & e^1{}_0 = 2{\cdot}4 \\
e^2{}_1 = -\cdot 12 & e^2{}_2 = -\cdot 08 & e^2{}_3 = -\cdot 14 & e^2{}_0 = \cdot 082 \\
e^3{}_1 = -\cdot 015 & e^3{}_2 = -\cdot 097 & e^3{}_3 = -\cdot 0111 & e^3{}_0 = \cdot 092 \\
 & & & e^0{}_0 = 2{\cdot}0
\end{array} \tag{5.3}$$

$e^0{}_1, e^0{}_2, e^0{}_3$ were calculated in the fashion of Muraskin (1971b) to make g_{00} a maximum at the origin. This set of data obeys the $R^i_{jkl} = 0$ integrability relations.

We have already argued in Muraskin (1973b) that we can get the same answer at all points for Γ^i_{jk}, g_{ij} if we consider a theory based on $e^\alpha{}_i$ with

$$\begin{aligned}
\frac{\partial e^\alpha{}_i}{\partial x^k} &= \Gamma^\alpha_{\beta\gamma} e^\beta{}_i e^\gamma{}_k \\
\Gamma^i_{jk} &= e_\alpha{}^i e^\beta{}_j e^\gamma{}_k \Gamma^\alpha_{\beta\gamma} \\
g_{ij} &= e^\alpha{}_i e^\beta{}_j g_{\alpha\beta}
\end{aligned} \tag{5.4}$$

with $\Gamma^\alpha_{\beta\gamma}$, $g_{\alpha\beta}$ constant. If we use data (5.1), (5.2) and (5.3), we find from the computer that $e^\alpha{}_i \to 0$ if we go far enough along the x-axis away from the origin. This is not the boundary condition $e^\alpha{}_i \to \delta^\alpha{}_i$ which we would like in order to enable us to introduce contravariant indices using the dual field $e_\alpha{}^i$. If $R^i{}_{jkl} \neq 0$ we may bring in $\bar{\Gamma}^\alpha{}_{\beta k}$, as in Muraskin (1973b), with the aim of satisfying $e^\alpha{}_i \to \delta^\alpha_i$. In the $R^i_{jkl} = 0$ situation the introduction of $\bar{\Gamma}^\alpha{}_{\beta k}$ is not a necessity. However, we never showed that it could not be introduced. We may recall that in Muraskin (1970) we were able to bring in $\bar{\Gamma}^\alpha{}_{\beta k}$ and $\Gamma^\alpha{}_{\beta\gamma}(x)$, $g_{\alpha\beta}(x)$ and still have $R^i{}_{jkl} = 0$. We shall assume this to be the case here in order that the boundary condition $e^\alpha{}_i \to \delta^\alpha_i$ be satisfied.

We perform a three-dimensional rotation about a coordinate axis using the formula

$$\Gamma'^{\alpha'}{}_{\beta'\gamma'} = a_\alpha{}^{\alpha'} a^\beta{}_{\beta'} a^\gamma{}_{\gamma'} \Gamma^\alpha_{\beta\gamma} \tag{5.5}$$

and

$$g'_{\alpha'\beta'} = a^{\alpha}{}_{\alpha'} a^{\beta}{}_{\beta'} g_{\alpha\beta} \tag{5.6}$$

We take, for example,

$$a^{\alpha}{}_{\alpha'}, \quad \begin{pmatrix} \cos\varphi & \sin\varphi & 0 & 0 \\ -\sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5.7}$$

This describes a rotation about the z-axis. We find that, unlike our findings in Muraskin (1973), the data is not invariant, although it is nearly invariant. All components except Γ^1_{20}, Γ^1_{30}, Γ^2_{30}, Γ^2_{10}, Γ^3_{10}, Γ^3_{20} remain unchanged. No $\Gamma^{\alpha}_{\beta\gamma}$ that was zero became non-zero. Thus, we have a structure that is similar to the original structure even though there is no invariance.

We have found, first of all, that g_{ij}, at all points considered, came out the same as the g_{ij} in the Muraskin (1973b) data. Thus, we get the same bounded particle for g_{ij} as we obtained before. This also suggests that $g_{ij} \to 0$ after we proceed sufficiently far from the origin. We have made long runs down the $\pm x$-axes and have also found that $\Gamma^i_{jk} \to 0$ if we go far enough down the axis. We note that even though g_{ij} is the same as in Muraskin (1973b) the Γ^i_{jk} are different.

In running down the $\pm x$-axes, we found as many as four turnabout points for components of Γ^i_{jk}. The largest number of turnabout points seen in our work previously was three.† Thus we have slightly greater oscillatory behavior than we had found previously. Also, we found that at $x = -381$ not all the components were decreasing in magnitude (one was still increasing). This situation was remedied by the time we got to $x \sim -700$. Thus, the monotonic behavior that we previously found down the axis far from the origin took much longer to achieve here. It is not clear if any of this is significant, but at any rate it would appear that we have some additional structure with the present data.

Also, it is not apparent whether this data is obtainable from a transformation on data that is invariant under three-dimensional rotations or not.

6. *Data* 5

In Muraskin (1973b) we pointed out that the choice of B (called C in Muraskin (1973b)) in equation (4.1) did not alter the values of g_{ij}. We could then, in fact, take $B = 0$ and we would still obtain a bounded particle in g_{00} with the same shape as when $B \neq 0$. But Γ^i_{jk} would be different for different B.

† The parameters used in the two cases were comparable. Note, it is possible that too large a grid could obscure turnabout points. This would be the case for both sets of data.

The question, therefore is whether there is some important role played by the B terms in the theory. Perhaps they are responsible for the lack of singularities in Γ^i_{jk}. Thus we have investigated the data of Muraskin (1973b) but with $B = 0$. The qualitative picture that emerges, however, was similar to the case of $B \neq 0$. Hence the role of B is not apparent. We still obtain $\Gamma^i_{jk} \to 0$ far down the axes. The integrability equations satisfied here are of the $R^i{}_{jkl} \neq 0$ variety. The oscillatory behavior is similar to the data of Muraskin (1973b).

7. *Data* 6

In this case we considered $A = C = 0$, $B \neq 0$ in equation (4.1). This by itself would give an exact solution to the field equations and does not help us to understand any possible need for the completely antisymmetric components. What we shall do is to take, in addition to the B terms, $\Gamma^0_{00} \neq 0$. Such data is similar to the data in Muraskin (1971b). In that paper all components having a zero index were zero except for Γ^0_{00}. We find that the present data satisfies the $R^i_{jkl} \neq 0$ integrability equations. We have taken $g_{\alpha\beta}$ to be given by equation (4.2). We found no maximum or minimum in g_{00} at the origin. We used the same $e^\alpha{}_i$ as in (4.3). The results show a tendency toward blow-up. All components of the field increase or decrease with an ever-increasing rate as we move down the axis. Note, in Muraskin & Ring (1972) we obtained a similar kind of result for the data proposed in Muraskin (1971b).

8. *Data* 7

In this section we ask whether it is possible to construct a theory based on g_{ij}, Γ^i_{jk} such that $\Gamma^i_{jk} \to 0$, $g_{ij} \to 0$ at infinity and for which $g \neq 0$ at the origin. Since $g_{ij} \to 0$ at infinity we do not have a non-singular g^{ij} at all points.

In $g = 0$ theory, $1/g$ is not a physically meaningful field, thus we do not introduce such a quantity. Similarly, if $g_{ij} \to 0$ at infinity the field $g^{ij}(x)$ will not be considered to be a physically meaningful variable, and will not be introduced into any of the basic field equations.†

We consider the following data

$$\Gamma^\alpha_{\beta\gamma} = \delta^\alpha_\beta \phi_\gamma + g_{\beta\gamma}\psi^\alpha + \delta^\alpha_\gamma \theta_\beta + g^{\alpha\sigma} B^\rho \epsilon_{\rho\sigma\beta\gamma} \tag{8.1}$$

with‡

$$\begin{aligned} \phi_\alpha &= \theta_\alpha = -\psi_\alpha = \Gamma^0_{00} \\ B_\alpha &= \text{const. } \phi_\alpha \end{aligned} \tag{8.2}$$

† The fact is, using the $g_{\alpha\beta}$, $\Gamma^\alpha_{\beta\gamma}$ data (8.1), (8.2) and (8.3), having $g \neq 0$, we can obtain a bounded particle solution of the equations $\Gamma^i{}_{jk;l} = 0$, $g_{ij;k} = 0$, for which the computer suggests $g_{ij} \to 0$, $\Gamma^i_{jk} \to 0$ at infinity.

‡ This data is consistent with an underlying structure that is invariant under three-dimensional rotations. This is because (8.1) and (8.2) can be obtained from a four-dimensional orthogonal transformation on a set of invariant data.

$g^{\alpha\beta}$ is defined at the origin and taken to be diag. (1, 1, 1, 1). Numerically, we chose $\phi_1 = \cdot 1$, $\phi_2 = \cdot 2$, $\phi_3 = \cdot 3$, $\phi_0 = \cdot 4$ and $B_\alpha = 2\phi_\alpha$. $g_{\alpha\beta}$ is taken to be (at the origin)

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8.3}$$

We take $e^\alpha{}_i$ to give a maximum in g_{00} (at the origin)

$$\begin{array}{llll} e^1{}_1 = \cdot 7 & e^1{}_2 = \cdot 62 & e^1{}_3 = \cdot 45 & e^1{}_0 = 1\cdot 5 \\ e^2{}_1 = -\cdot 12 & e^2{}_2 = -\cdot 08 & e^2{}_3 = -\cdot 14 & e^2{}_0 = \cdot 082 \\ e^3{}_1 = -\cdot 015 & e^3{}_2 = -\cdot 097 & e^3{}_3 = -0\cdot 111 & e^3{}_0 = \cdot 092 \\ & & & e^0{}_0 = 1\cdot 0 \end{array} \tag{8.4}$$

$e^0{}_1$, $e^0{}_2$, $e^0{}_3$ are calculated as in Muraskin (1971b). Now (8.1) and (8.2) lead to a set of numbers for $\Gamma^\alpha_{\beta\gamma}$. Together with the numbers (8.3) and (8.4) we find that the $R^i_{jkl} \neq 0$ integrability equations are satisfied and $\Gamma^i_{jk} \to 0$, $g_{ij} \to 0$ is observed far from the origin in our computer studies.

It is assumed that there exists a set of $g_{\alpha\beta}(x)$, $\Gamma^\alpha_{\beta\gamma}(x)$, $\bar{\Gamma}^\alpha{}_{\beta k}$ so that $e^\alpha{}_i$ satisfies $e^\alpha{}_i \to \delta^\alpha_i$ far from the origin in order to justify the contravariant index in Γ^i_{jk}.

The computer studies show that the g_{00} particle is bounded. We have not obtained any additional oscillatory behavior than in Muraskin (1973b).

The data above appears to lead to a bounded particle without requiring $g = 0$. The computer work serves to illustrate that $\Gamma^i_{jk} \to 0$ at infinity can occur when $g_{ij} \to 0$ at infinity without requiring that the auxiliary conditions of Muraskin (1972c) be satisfied.†

9. *Data* 8

We have shown that we can obtain a bounded particle even when the completely antisymmetric components were zero as in Data 5. Since these components are not essential for a bounded particle, this suggests that we may be able to get away with more simple kinds of theories. The most simple theory would involve the case in which $g_{\alpha\rho}\Gamma^\alpha_{\beta\gamma}$ is completely symmetric in all indices as such a situation can be related to a single-field variable. To see this point, we write down the field equation for g_{ij}

$$\frac{\partial g_{ij}}{\partial x^k} = \Gamma^t_{ik} g_{tj} + \Gamma^t_{jk} g_{it} \equiv \Gamma_{jik} + \Gamma_{ijk} \tag{9.1}$$

† The invariants in Muraskin (1972c) would involve multiplication of infinity by zero at space-time infinity, thus the conclusions of that paper would not apply.

Since Γ_{ijk} is to be completely symmetric, we have

$$\frac{\partial g_{ij}}{\partial x^k} = \frac{\partial g_{ik}}{\partial x^j} \tag{9.2}$$

This equation is identically satisfied if

$$g_{ij} = \frac{\partial^2 \phi}{\partial x^i \partial x^j} \tag{9.3}$$

g_{ij} is symmetric in i and j as it should be. From (9.1) we then obtain

$$\Gamma_{ijk} = \frac{1}{2} \frac{\partial^3 \phi}{\partial x^i \partial x^j \partial x^k} \tag{9.4}$$

which is symmetric in all indices as required. Thus, g_{ij} and Γ_{ijk} can be built up from a single variable, ϕ.† It is not clear that $\Gamma^i_{jk;l} = 0$ then leads to solutions that are everywhere finite. This is something that we shall investigate.

A set of $g_{\alpha\rho}\Gamma^\alpha_{\beta\gamma}$, which is completely symmetric in all indices, is given by

$$\begin{aligned} \Gamma^1_{11} &= a = -\cdot 2 \\ \Gamma^2_{22} &= b = \cdot 3 \\ \Gamma^3_{33} &= c = \cdot 6 \\ \Gamma^0_{00} &= d = -\cdot 7 \end{aligned} \tag{9.5}$$

with $g_{\alpha\beta}$ given by (8.3). This data obeys $R^i_{jkl} = 0$ integrability. We obtain a minimum in g_{00} at the origin using the same $e^\alpha{}_i$ as in (8.4). $e^0{}_1, e^0{}_2, e^0{}_3$ were calculated in the manner of Muraskin (1971b). The results from the computer show no bound in g_{00} developing. A note of caution should be interjected at this point. When some components of the field continue to increase or decrease at an ever-increasing rate after a reasonably long run, this does not constitute proof of a singularity developing since the magnitudes associated with the particles may be extremely large. Below we give representative data.

	g_{00}	Γ^1_{00}	Γ^2_{31}
$x = 0$	8·02	1·78	−·069
$x = 1$	8·51	2·29	−·053
$x = 2$	10·2	2·94	−·041
$x = 3$	14·0	3·85	−·032
$x = 4$	22·6	5·30	−·025
$x = 5$	46·7	8·07	−·017
$x = 6$	164·5	15·7	−·004
$x = 6·98$	11,364·0	134·4	·145

† This scalar function is not constructed from products of g_{ij}, Γ^i_{jk}, $\partial_i\Gamma^m_{jk}$, $\partial_k g_{ij}$, as is the scalar functions discussed in Muraskin (1972c).

A turnabout was seen in some components of Γ^i_{jk} during this run. However, the data appears to us to suggest that a singularity could be developing.

10. *Data* 9

In Muraskin (1970) and Muraskin & Clark (1970) we introduced a set of data called particle *A* and particle *B*. This sort of data exhibited a reflection symmetry about the origin for many of the components of the field to computer accuracy. Thus, this set of data shows that the field is capable of describing symmetric configurations. We have found that after long runs down the axes, no bound showed up for g_{00}. There was also a plane through the origin on which components such as Γ^1_{23} did not change at all. We are, therefore, rather suspicious of this data.

11. *Data* 10

In Muraskin (1972b) we introduced a set of data that obeys $R^i{}_{jkl} \neq 0$. We were unable to arrange the quantities appearing in equation (11) of Muraskin (1971b) to be non-zero. Thus, no maximum or minimum in g_{00} was found at the origin. Running down the x-axis we found all components of the field growing larger and larger in magnitude, suggesting a blow-up.

12. *Conclusions*

We first of all see that there are many solutions of the integrability equations, of which we have discovered a small number.

We have found a bounded particle solution that has only slightly more structure than we had obtained previously. We had hoped for a bounded particle immersed in an 'agitated' vacuum.

We must also keep in mind two natural limitations in our computer program. First, the distances between particle systems may be enormous. Thus, the data of Muraskin (1973b) or data 4 or data 7 (as an example) cannot be ruled out as a possibility, even though we have not found an agitated vacuum. It may be that one has to make exceedingly long runs before additional structure shows up. The second limitation is that the value of field components associated with a particle may be many orders of magnitude larger than the environment. Thus, data 8 could conceivably still describe a bounded particle.

We are not against making runs on the computer of considerable longer duration than we have up to now. However, we would like to first justify to ourselves that there is a reasonable chance of something worthwhile coming out before such an attempt is made. The criterion we are looking for (and we have to admit that we may be mistaken) is the presence of an agitated vacuum, or at least a vacuum with considerably more structure than we have seen.

We have noted from data 1 that sines and cosines are present in the theory; we have seen that a bounded particle is present; we have seen that the apparent absence of singularities is present; we have seen that the trends toward natural boundary conditions are present; so perhaps a better set of data does exist.

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